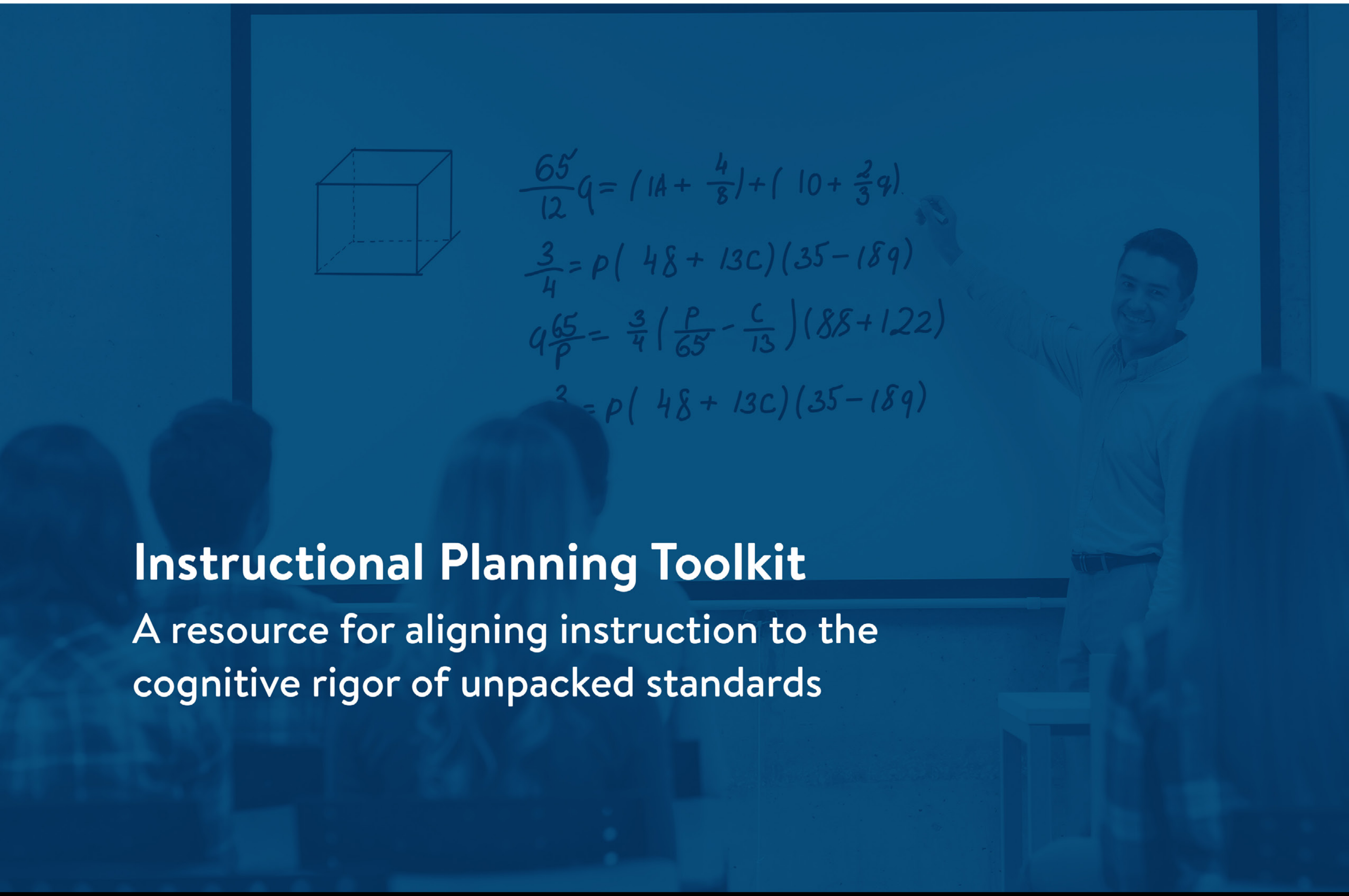
College and Career Ready Standards



Instructional Planning Toolkit

A resource for aligning instruction to the cognitive rigor of unpacked standards

Mathematics

Algebra

LEAD180®

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JUMP DIRECTLY TO THE STANDARDS

LEAD180 FRAMEWORK

Let's face it: teachers and school leaders are in the midst of a big, ever-changing shift in the way they educate students. Finding the perfect balance that works for school districts and for individual schools, while improving teacher and student performance, has been quite the endeavor. Evidence from the first few years of Next Generation Assessments shows that students are struggling to meet college and career-ready standards (CCRS), regardless of how many hours teachers and school leaders spend preparing them.

With School Leadership Solutions' **LEAD180**, teachers and school leaders can create sustainable change within classrooms, leadership teams, and schools that will last for years to come, regardless of what changes in the education world.

Former principal Scott Neil developed **LEAD180** as a way to transform individual schools and improve student performance, based on his needs as an instructional leader. After using this system consistently in his own schools for several years, he saw dramatic increases in student and teacher performance across multiple schools. Since then, he's developed it further to help schools and districts throughout the country navigate the ever-changing requirements of CCRS and transform teacher and student achievement semester after semester.

LEAD180 focuses on three guiding core concepts that make it successful:

PEOPLE

ALIGNMENT

SUPPORT

PEOPLE

First and foremost, sustainable change must come from within. As educational institutions molding the minds of students, schools must work with a diverse demographic on a regular basis. That means school leaders must be people focused, with a high degree of emotional intelligence. LEAD180 serves as a guide to help schools create a culture of success by building the self-efficacy of parents, teachers, and students. Our program aims to strengthen schools' collective efficacy by providing evidence tied to goals and higher expectations. LEAD180 structures and protocols provide ongoing feedback to teachers and students based on evidence that they can succeed with support and increased effort, by changing their beliefs and expectations, and by setting goals and incorporating evaluative information. The LEAD180 Principles of Performance constantly focus on producing outcomes through methods that create stronger connections with school staff and administrators.

ALIGNMENT

Data is useless unless it is aligned with standards and provides students with accurate feedback. LEAD180 does just that, giving feedback to both students and teachers that is already aligned to the rigor of CCRS, teacher instruction, and the state's curriculum and assessments. LEAD180's Four Steps to Curriculum Alignment method sees that all facets of teaching, learning, and feedback are aligned to the cognitive rigor of CCRS. The Four Steps to Curriculum Alignment ensure that what is being taught and when it will be taught is mapped out in advance of the school year. This allows teachers to plan and design their instruction and assessments based on the appropriate rigor level of the intended standard or target.

SUPPORT

Schools cannot be successful without ongoing support for student and professional learning. LEAD180 helps you with every step by offering solutions for ongoing professional and student learning so schools can create effective learning structures. LEAD180 guides schools through the process of designing a multi-tiered approach to professional learning for teachers. Support for student learning is provided as part of an established data-driven culture focused on providing corrective instruction to meet the individual needs of students. As teachers focus on educating their students, LEAD180 provides coaching to support instruction and learning while giving teachers time to reflect and provide feedback to one another.

INSTRUCTIONAL PLANNING TOOLKIT

You have in your possession a powerful tool that can help solve the achievement gap within CCRS. The Instructional Planning Toolkit (IPT) is at the heart of the LEAD180 core concept of alignment. It acts as a blueprint to help educators required to align with CCRS in English language arts (ELA) and mathematics successfully plan their instruction.

Simply put, our IPT makes preparing for the rigor of CCRS much easier and saves precious time by pulling essential planning tools that you need into one, easy-to-reach place. The digital IPT—an interactive PDF that is compatible with the free Adobe Acrobat Reader app—works with tablets and smartphones. Every piece of information you need for any standard and target aligned to CCRS will be a touch away on your device.

After using the IPT, teachers will better understand and plan more effectively for the rigor and intent of each unpacked ELA and mathematics CCRS.

Purpose of the Instructional Planning Toolkit

The IPT organizes what students should know and are expected to do within each CCRS. Required standards have been broken down by Depth of Knowledge (DOK) levels and aspects of rigor (mathematics) and divided again into the specific instructional targets for each grade level. Outlining the details of each standard in this way provides targeted support to teachers as they develop their curriculum maps and unit and lesson plans.

The IPT provides a range of suggested questions and teaching strategies that are differentiated to help all students move toward mastery of each standard within each subject area. The IPT consists of a compilation of preexisting materials in the public domain from the Common Core State Standards for ELA/Math and the Appendix from the Common Core State Standards Initiative. A team of content experts developed strategies, question stems, vocabulary and other useful resources in an effort to reduce the time it takes teachers to plan for instruction that is aligned to CCRS.

What Is DOK?

At the core of CCRS is the need to raise the level of rigor for all students. States' adoption of more stringent standards has resulted in an increased focus on rigor. Schools need to make informed decisions as they develop curriculum and assessments and plan instruction aligned to the higher levels of cognitive demand required by CCRS.
Webb's Depth of Knowledge (DOK) is a key resource that educators can use to analyze the cognitive demand,

or complexity, intended by the standards. Developed by Dr. Norman Webb in 1997, the DOK initially served as a process and criterion for analyzing the alignment between standards and test items in standardized assessments. It has become an effective tool for reviewing curriculum for alignment as well. The DOK categorizes tasks by different levels of cognitive demand—or depth of knowledge—required to successfully complete the task. Using the DOK levels with the instructional targets within the IPT will help solve the mystery for teachers as they plan for the levels of rigor within each standard. The table below outlines the Webb DOK levels:

DOK 1

Recall and Reproduction

Curricular elements that fall into this category involve basic tasks that require students to recall or reproduce knowledge and/or skills. The subject matter content at this particular level usually involves working with facts, terms, and/or properties of objects.

DOK 2

Skills and Concepts

Includes the engagement of some mental processing beyond recalling or reproducing a response. Elements of a curriculum that fall into this category involve working with or applying skills and/or concepts to tasks related to the field of study in a laboratory setting.

DOK 3

Short-Term Strategic Thinking

Items falling into this category demand a short-term use of higher-order thinking processes, such as analysis and evaluation, to solve real-world problems with predictable outcomes. Stating one's reasoning is a key marker of tasks that fall into this particular category.

DOK 4

Extended Thinking

Curricular elements assigned to this level demand extended use of higher-order thinking processes such as synthesis, reflection, assessment, and adjustment of plans over time. Students are engaged in conducting investigations to solve real-world problems with unpredictable outcomes.

A Focus on Results Rather Than Means

By emphasizing required achievements, CCRS gives teachers, curriculum developers, and states the freedom to determine how those goals should be reached and which other topics need to be addressed. For example, CCRS doesn't cover such details as a particular writing process or the full range of metacognitive strategies that students may need to monitor and direct their thinking and learning. Therefore, teachers have the freedom to provide students with whatever tools and knowledge they think is most helpful to meet the goals set out in CCRS. Our IPT provides guidance for teachers, collaborative teams, schools, and districts on how to plan for mastering CCRS.

Distribution of Literary and Informational Passages by Grade in the 2009 NAEP

Mathematics Framework

CCRS aims to align instruction with the framework below so that more students can meet college and career requirements. The IPT provides guidance by listing suggested literary and informational texts for each unpacked standard.

	Grade 4	Grade 8	Grade 12
Number properties and operations	40%	20%	10%
Measurement	20%	15%	30%
Geometry	15%	20%	
Data analysis, statistics, and probability	10%	15%	25%
Algebra	15%	30%	35%

Source: National Assessment Governing Board. (2015).

Key Terms Related to the College and Career Ready Standards and Progressions

The Instructional Shifts in Math

Greater focus on fewer topics: The CCRS calls for greater focus in mathematics. Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom. This means focusing deeply on the major work of each grade as follows:

- **In grades K–2:** Concepts, skills, and problem solving related to addition and subtraction
- **In grades 3–5:** Concepts, skills, and problem solving related to multiplication and division of whole numbers and fractions
- **In grade 6:** Ratios and proportional relationships, and early algebraic expressions and equations

- **In grade 7:** Ratios and proportional relationships, and arithmetic of rational numbers
- **In grade 8:** Linear algebra and linear functions

This focus will help students gain strong foundations, including a solid understanding of concepts, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the classroom.

Coherence: Linking topics and thinking across grades. Mathematics is not a list of disconnected topics, tricks, or mnemonics; it is a coherent body of knowledge made up of interconnected concepts. Therefore, the standards are designed around coherent progressions from grade to grade. Learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. For example, in 4th grade, students must "apply and extend previous understandings of multiplication to multiply a fraction by a whole number" (Standard 4.NF.4). This extends to 5th grade, when students are expected to build on that skill to "apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction" (Standard 5.NF.4). Each standard is not a new event, but an extension of previous learning. Coherence is also built into the standards in how they reinforce a major topic in a grade by utilizing supporting, complementary topics.

Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity. Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

- ***Conceptual understanding:*** The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.
- ***Procedural skills and fluency:*** The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.
- ***Application:*** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Source: National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

The IPT unpacks the standards and organizes them into individual learning targets. The learning targets have been organized by both the three aspects of rigor and DOK levels to help provide schools and teachers with the information they need to design lessons, instruction, tasks, and assessments with the specific levels of rigor in mind.

National Council of Teachers of Mathematics: Math Teaching Practices (Teacher Practice)

The National Council of Teachers of Mathematics has proposed eight teaching practices that should be consistent in every mathematics lesson to ensure alignment to CCRS.

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning:
Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving:
Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations:
Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse:
Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions:
Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding:
Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning Mathematics:
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking:
Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice (Student Practice)

The Standards for Mathematical Practice are the standards that students are expected to meet at each grade level (or subject area in high school). The standards are broken into eight domains across grades and subject areas:

1 Make Sense of Problems and Persevere in Solving Them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason Abstractly and Quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct Viable Arguments and Critique the Reasoning of Others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with Mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use Appropriate Tools Strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to Precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look For and Make Use of Structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as 2+7.

They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look For and Express Regularity in Repeated Reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation (y-2) / (x-1) = 3. Noticing the regularity in the way terms cancel when expanding (x-1)(x+1), $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series.

Source: National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author.

Focus and Coherence in Instruction and Assessment

- **FOCUS:** CCRS call for greater focus in mathematics. Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom.

  There are two aspects of focus:
  1. Focusing where the standards focus
  2. Focusing deeply on the major work of the grade

- **COHERENCE:** Linking topics and thinking across grades. CCRS are designed to be coherent so that they build on each other from year to year in a logical progression. The curriculum is marked by effective movement from less sophisticated topics in the lower grades to more sophisticated ones as students gain deeper understanding of the math.

Although CCRS provides specific expectations in mathematics, individual standards don't have to be a separate focus for instruction and assessment. Often, several standards can be addressed by a single rich task. Our IPT provides guidance by suggesting paired standards to build focus and coherence in instruction and assessment.

Conclusion

We hope you find this toolkit a valuable resource that helps you align your instruction and save valuable planning time. The **IPT** is not intended as an end-all-be-all solution for effective teaching methods, but it can be the end of your CCRS worries and the beginning of improving your students' education.

ORGANIZATION AND USE OF THE INSTRUCTIONAL PLANNING TOOLKIT (IPT)

MATHEMATICS Grade 3 (1)

3.OA | 3.OA.A.1 | 3.OA.A.2 | 3.OA.A.3 | 3.OA.A.4 | 3.OA.B.5 | 3.OA.B.6 | 3.OA.C.7 | 3.OA.D.8 | 3.OA.D.9 | 3.NBT | 3.NBT.A.1 | 3.NBT.A.2 | 3.NBT.A.3 | 3.NF | 3.NF.A.1 | 3.NF.A.2 | 3.NF.A.3 | 3.MD | 3.MD.A.1 | 3.MD.A.2 | 3.MD.B.3 | 3.MD.B.4 | 3.MD.C.5 | 3.MD.C.6 | 3.MD.C.7 | 3.MD.D.8 | 3.G | 3.G.A.1 | 3.G.A.2

STANDARD 3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	2
Big Idea	Developing understanding of multiplication and division and strategies for multiplication and division within 100.	3
Cluster	Represent and solve problems involving multiplication and division.	4
Academic Vocabulary	• **Factor** — a number that is multiplied by another number to find an answer or product • **Multiplication/multiply** — an operation that can be defined by repeated addition of the same value • **Product** — the solution when multiplying values	5
Standard Cognitive Complexity	Content Complexity Rating: Level 1	6
Standard Progression	Second-Grade Coherence — 2.NBT.2, 2.OA.3, and 2.OA.4 Third-Grade Coherence — 3.OA 2–5 and 3.MD.7	7
Mathematical Practices and Suggested Questions to Develop Mathematical Thinking	MP1 **Construct viable arguments and critique the reasoning of others** • How did you decide what the problem was asking you to find? MP2 **Attend to precision** • Explain how you know your answer is the solution to the problem	8
Essential Question Stems	• How can we interpret multiplication as repeated addition? • What is another problem that can represent the equation? • What does the product represent in the equation?	9

GRADE-LEVEL STANDARD UNPACKED

Aspects of Rigor	Instructional Targets	
Conceptual Understanding	• Interpret products of whole numbers as a total number of objects in a number of group. (DOK 1)	10

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MATHEMATICS Grade 3

3.OA | 3.OA.A.1 | 3.OA.A.2 | 3.OA.A.3 | 3.OA.A.4 | 3.OA.B.5 | 3.OA.B.6 | 3.OA.C.7 | 3.OA.D.8 | 3.OA.D.9 | 3.NBT | 3.NBT.A.1 | 3.NBT.A.2 | 3.NBT.A.3 | 3.NF | 3.NF.A.1 | 3.NF.A.2 | 3.NF.A.3 | 3.MD | 3.MD.A.1 | 3.MD.A.2 | 3.MD.B.3 | 3.MD.B.4 | 3.MD.C.5 | 3.MD.C.6 | 3.MD.C.7 | 3.MD.D.8 | 3.G | 3.G.A.1 | 3.G.A.2

Procedural Skills and Fluency	• Find the product of multiple groups of objects. (DOK 1)
Application	• N/A

Suggested Instructional Strategies (11)

On level	The structure of an equal-group or array multiplication word problem has a specific structure. Students need to recognize this structure and interpret the problem by representing it with a multiplication equation. In return they also need to write an equal-group or array word problem using the same structure that is a situation represented by the problem. Students will need practice reading and writing these problems.	
Enrichment	As students begin to see the process of writing the equations and word problems, they can apply this to values greater than 10 by 10. To stay within the grade-level criteria, students can represent the missing products with a symbol.	
Ideas to Support the Standard	Students need to show that they can identify the structure of the multiplication word problem, a skill they will use as they progress through the operations. Although concrete models are not required, students may need them to better process their understanding.	12

EXPLANATIONS AND EXAMPLES (13)

	When interpreting situations of multiplication, students will need exposure to numerous problems that demonstrate multiplication as equal groups. They should notice a pattern within the problems of key phrases that indicate multiplication. Often problems use the example of "groups of," but they will notice that it can be any word that indicates that there are equal groups, such as boxes, packages, containers, bags, trays, and more. All of these key phrases build the structure of equal group problems. Students who understand the structure will apply this to other standards both in third grade and subsequent grades. This understanding allows students to find a total number of objects not by counting them individually, but by counting them quickly in groups.	
Common Misconception	Focusing on a single key word will cause confusion. Students will see the word of and automatically think they are being asked to do multiplication, regardless of the question being posed. Use of specific situations and the structure of equal-group problems will give students a solid foundation in recognizing equal-group problems. In the same respect, students have learned in previous years that *altogether* and *in all* are key words for addition. This concept is correct because they are repeatedly adding but as an equal number of groups, not as single amounts. (Example: not 6 +7 but 6×7.)	14

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1. Common core grade level and domain. A domain is a group of related standards.
2. Grade level specific standard. Students advancing through the grades are expected to meet each grade year's grade level specific standards, retain or further develop skills and understandings mastered in preceding grades, and work steadily toward meeting the more general expectations described by the CCR standards.
3. A big idea is an overarching idea that connects the standards.
4. A cluster is a summary of related standards within a domain.
5. Academic vocabulary is specific to the standard.
6. Overall complexity level for the standard.
7. Standard Progression. The standard progression is a suggested list of coherent standards within a grade level or from previous grade levels.
8. Mathematical Practices. These are standards that describe a list of practices and procedures that are needed to master mathematics. The questions for each mathematical practice are listed to assist the learners in building their own deep understanding.
9. Essential questions to guide unit development and instruction.
10. Aspects of Rigor and Instructional Targets. Rigor is one of the three shifts of instruction described in CCRS. There are three levels of rigor; Conceptual Understanding, Procedural Skills and Fluency, and Application. Conceptual understanding has the learner "seeing" the mathematics. Procedural skills and fluency is the learner building processes that build speed and accuracy in calculations. Application has the learner correctly applying their mathematical knowledge to solve problems. The instructional targets decompose the standard into the skills described in the standard. The instructional targets are labeled with the DOK or cognitive ability for each target.
11. Suggested Instructional Strategies. The suggested instructional strategies are broken into traits of three types of learners. These strategies can be used to plan for instruction, as well as, monitor the progress of learners.
12. Ideas to support are additional examples that are generally not a part of the standards but that assist in the learner mastering the standard.
13. Explanations and Examples. The explanations are what teachers should expect when teaching the content. One or more sample tasks are provided to model a rigor of the standard. The solution is provided to model a sample of what a student might provide as a response to the task. The student processes should be the focus when providing feedback from the task.
14. Common misconceptions. These are the most common mistakes seen when the learner is completing tasks for a specific standard or concept. These are not the only misconceptions. These can be used when planning when choosing examples for lessons, as well as, creating reteaches.

HIGH SCHOOL:
ALGEBRA » SEEING STRUCTURE IN EXPRESSIONS

Standards in this domain:

- CCSS.MATH.CONTENT.HSA.SSE.A.1
- CCSS.MATH.CONTENT.HSA.SSE.A.2
- CCSS.MATH.CONTENT.HSA.SSE.B.3
- CCSS.MATH.CONTENT.HSA.SSE.B.4

INTERPRET THE STRUCTURE OF EXPRESSIONS

CCSS.MATH.CONTENT.HSA.SSE.A.1
Interpret expressions that represent a quantity in terms of its context.*

CCSS.MATH.CONTENT.HSA.SSE.A.1.A
Interpret parts of an expression, such as terms, factors, and coefficients.

CCSS.MATH.CONTENT.HSA.SSE.A.1.B
Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

CCSS.MATH.CONTENT.HSA.SSE.A.2
Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

INTERPRET THE STRUCTURE OF EXPRESSIONS

CCSS.MATH.CONTENT.HSA.SSE.B.3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

CCSS.MATH.CONTENT.HSA.SSE.B.3.A
Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.MATH.CONTENT.HSA.SSE.B.3.B
Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

CCSS.MATH.CONTENT.HSA.SSE.B.3.C
Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

CCSS.MATH.CONTENT.HSA.SSE.B.4
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
A.APR.D.6
A.APR.D.7
A.CED
A.CED.A.1
A.CED.A.2
A.CED.A.3
A.CED.A.4
A.REI
A.REI.A.1
A.REI.A.2
A.REI.B.3
A.REI.B.4
A.REI.C.5
A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

STANDARD A.SSE.A.1

Interpret expressions that represent a quantity in terms of its context.

A. Interpret parts of an expression, such as terms, factors, and coefficients.

B. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

Big Idea

Every expression can be interpreted based on its components. The structure of the expression can tell us something about the context to which it refers.

Cluster

Interpret the structure of expressions.

GRADE-LEVEL STANDARD UNPACKED

Academic Vocabulary

- **Expression** — Numbers, symbols grouped together to show the value of something
- **Factor** — A number that is multiplied by another number to find a product

Standard Cognitive Complexity

Content Complexity Rating: Level 2

Standard Progression

6th Grade Coherence: 6.EE.2.b
7th Grade Coherence: 7.EE.2
High School Coherence: A-SSE.2

Mathematical Practices and Suggested Questions to Develop Mathematical Thinking

MP7

Look for and make use of structure

- What must be true about the value of the expression 5 – (x - 4)2 for all real values of x?
- How can you interpret (x + 3)(x – 4) as a product?

Essential Question Stems

How do the variables in this expression relate to each other and to the context it describes?

Aspects of Rigor	Instructional Targets
Conceptual Understanding	• Make sense of the multiple factors in an expression by explaining the meaning of the individual parts. (DOK 2)
Procedural Skills and Fluency	• Identify the different parts of the expression. (DOK 1) • Decompose expressions. (DOK 1)
Application	• Explain the meaning of the different parts of an expression within the context of a problem. (DOK 2)

Suggested Instructional Strategies

On level

Students should be familiar with the terms factor, coefficient, product, sum, etc. from middle school. Students are probably less familiar with chunking (seeing parts of an expression as a single object). By the end of Algebra 1, students should be fluent in transforming expressions and chunking. As students learn multiple ways to solve quadratic equations (A.REI.4), they should see the parts of the quadratic expression as independent entities. For example, looking at $(x - 4)(x + 2) = 0$, students should see this as two factors, and using the zero product property, recognize that when two numbers are multiplied to equal zero, at least one of the numbers has to be zero. Therefore, either $x - 4 = 0$, or $x + 2 = 0$, so the solution of the equation has to be 4 or -2.

As students graph quadratic functions (F.IF.7), they should also see that in the function $f(x) = (x - 5)^2 + 4$, for all real values of x, the quantity $(x - 5)^2$ will always be positive. Therefore, the minimum value of the graph will be 4, which will take place when $(x - 5)^2 = 0$, and $x = 5$. Therefore, the vertex of the graph must be at the point (5, 4).

As students master the concept of chunking, they should be using it within context. In a falling object context, students should be able to recognize that in the expression --16(x + 1)(x - 6), the (x – 6) is the only factor that will provide them with a positive x-intercept to determine when the object hits the ground.

Intervention

The extensive work with quadratics in Algebra 1 allows students to have hands-on practice with expressions with multiple components. Students need to have practice finding many possible solutions to $y = (x - 5)^2 + 4$. As students look at their self-created tables, they can answer the questions of what the lowest y-value is and why there cannot be a value below 4. It may help for students to begin with $y = x^2 + 4$ and then extend to a quantity being squared.

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
A.APR.D.6
A.APR.D.7
A.CED
A.CED.A.1
A.CED.A.2
A.CED.A.3
A.CED.A.4
A.REI
A.REI.A.1
A.REI.A.2
A.REI.B.3
A.REI.B.4
A.REI.C.5
A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

Students can do similar work with tables as they consider possible solutions for y = (x – 4)(x + 2) and what values for x would make the y-value 0. Again, it might be useful to start with y = a*b and what values of a and b make y = 0, or even 0 = 4*b.

Enrichment

As students advance through Algebra 1, they should consider generalizing structure to functions based on parent functions of y = x, $y = x^2$, and $y=a*b^x$. Students can extend the thinking of this to connect to F.BF.3 and how the structure of expressions such as √(x+5)-4 tells us about possible values of the expression for all values of x. Within context, exponential equations and compounded interest will provide great examples where students have to interpret the base with regards to the exponent to discover the interest rate.

Ideas to Support the Standard

Students should constantly be making connections between the Functions and Algebra conceptual categories. Within the Algebra conceptual categories, the domains cannot be taught in silos. As students are learning about the structure of expressions, they should also be using these expressions within equations and functions to graph or solve. Clear connections can be made to F.BF.3, F.IF.4&7, and A.REI.4.

EXPLANATIONS AND EXAMPLES

Sample Task:
A scientist provided a student with a sample of 1,500 bacteria. He noticed that the number of bacteria in the sample after t days can be modeled by the equation $P=1{,}500*5^t$. In this equation, what does 5^t represent?

A. The number of bacteria increases by 5 bacteria each day.
B. The number of bacteria increases by t bacteria after 5 days.
C. The number of bacteria increases by a factor of 5 each day.
D. The number of bacteria increases by a factor of t each day for 5 days.

Solution: C
Source: https://parcc-assessment.org/wp-content/uploads/2018/01/MathReleasedItems/Algebra-I-Item-Set-2017-1-1.pdf

A rectangular garden has a length that is 3 feet longer than its width. Let w represent the width of the garden, in feet. The entire garden is surrounded by a 2-foot-wide cement walk-way. What does the expression (w + 4)(w + 7) represent in this context?

Solution: The total area of the garden and walkway.
Source: https://parcc-assessment.org/content/uploads/released_materials/02/Algebra_1_EOY_Item_Set_0.pdf

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
A.APR.D.6
A.APR.D.7
A.CED
A.CED.A.1
A.CED.A.2
A.CED.A.3
A.CED.A.4
A.REI
A.REI.A.1
A.REI.A.2
A.REI.B.3
A.REI.B.4
A.REI.C.5
A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

Common Misconceptions

Students frequently incorrectly believe that $(a + b)^2 = a^2 + b^2$. It is important for students to see multiple numeric examples to prove that this is untrue. Students often have the same misconception that they can "distribute" square roots, exponents, or absolute value to the quantities defined. For example, $|a+b| \neq |a|+|b|$ for all values of a and b. Giving students examples of a = 4 and b = -4 will help them understand this.

STANDARD A-SSE.A.2

Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Big Idea

Every expression can be interpreted based on its components. The structure of the expression can tell us something about the context to which it refers.

Cluster

Interpret the structure of expressions.

GRADE-LEVEL STANDARD UNPACKED

Academic Vocabulary

- **Expression** — Numbers, symbols grouped together to show the value of something
- **Factor** — A number that is multiplied by another number to find a product

Standard Cognitive Complexity

Content Complexity Rating: Level 2

Standard Progression

7th Grade Coherence — 7.EE.2
High School Coherence — A-APR.3, A-SSE.3, A-APR.4-6

Mathematical Practices and Suggested Questions to Develop Mathematical Thinking

MP7

Look for and make use of structure

- What underlying structure could allow me to rewrite this in a more manageable way?
- What relationships are there in this expression that look similar to patterns that we have discovered before?
- How can I determine that the expression I have created is equivalent to the original expression?

Essential Question Stems

- Could you relate this expression to a real-life scenario?

Aspects of Rigor

Instructional Targets

Conceptual Understanding	• N/A

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
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A.REI.B.3
A.REI.B.4
A.REI.C.5
A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

Procedural Skills and Fluency	• Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms. (DOK 2) • Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. (DOK2) • Simplify expressions including combining like terms, using the distributive property, and other operations with polynomials. (DOK 1)
Application	• N/A

Suggested Instructional Strategies

On level

As a crosscutting standard, in Algebra 1, students should focus on the structure of numerical expressions and basic polynomial expression. In Algebra 2, students can extend their understanding of polynomials and also deal with rational and exponential expressions.

Students will see structure in Algebra 1 as they begin to factor and then advance to factoring by grouping and using difference of squares. This gets them ready to use this structure within more complicated expressions in Algebra 2. In order for students to consider end behavior and points of discontinuity of rational function, they must first be able to factor the polynomials that make up the numerator and denominator.

Intervention

If students are having difficulty with factoring in Algebra 1, allow them to discover their own patterns by providing them with multiple binomial and trinomial expressions that are multiplied together. As students gain experience with multiplying polynomials, they will begin to recognize consistent patterns. Then ask students to factor polynomials that follow a similar structure, where students have to do the operations they have been practicing in reverse.

Enrichment

Students can be introduced to more advanced structural patterns such as sum and difference of cubes or using substitution to factor.

Extension questions:
Rewrite $x+8\sqrt{x}+15$ as a product of two factors.
Solution: $(\sqrt{x}+3)(\sqrt{x}+5)$

Ideas to Support the Standard

The standards purposefully avoid the word "simplify." As different forms of an expression can shed light on a problem or can assist in the next step of a problem, the word simplify can be confusing. Successful students will be able to flexibly identify and create different forms of an expression that will aid in whatever the next step of the problem is, all while maintaining equivalence.

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
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A.REI.B.4
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A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

EXPLANATIONS AND EXAMPLES

Complete the sentence by selecting the correct phrase:

The expression $(a^2)^2 - (b^2)^2$ is an example of:

- ☐ a system of equations
- ☐ a polynomial of degree 2
- ☐ a difference of squares
- ☐ a completely factored expression
- ☐ an exponential function

Which of the listed expressions are equivalent to $(a^2)^2 - (b^2)^2$:

- ☐ A. $a^4 - b^4$
- ☐ B. $a^4 + b^4$
- ☐ C. $(a^2+b^2)(a^2-b^2)$
- ☐ D. $(a^2+b^2)(a+b)(a-b)$
- ☐ E. $(a-b)^4$

Solution:
Part A: Difference of Squares
Part B: A, C, D

Source: https://parcc-assessment.org/content/uploads/released_materials/02/Algebra_1_EOY_Item_Set_0.pdf

Which expressions are true for all values of x? (Select all that apply.)

A. $3^{(2-x)}=3^2-3^x$
B. $3^{(x+2)}=9(3^x)$
C. $(3^x)^2=(3^2)^x$
D. $9^{x+2}=3^{2x+4}$
E. $27^x=(3^x)^3$

Solution: B,C,D,E

Source: https://parcc-assessment.org/content/uploads/released_materials/03/Algebra_2_PBA_Item_Set.pdf

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
A.APR.D.6
A.APR.D.7
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A.CED.A.3
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A.REI.B.3
A.REI.B.4
A.REI.C.5
A.REI.C.6
A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

Find a value for a, a value for k, and and a value for n so that

$$(3x+2)(2x-5) = ax^2 + kx + n.$$

Solution: a = 6, k = -10, n = -11

Source: https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/87

Common Misconceptions

It is common for students to create rules for simple expressions and expect them to follow for more difficult expressions. Using the term FOIL (First, Outer, Inner, Last) seems like an easy enough way for students to learn to distribute when multiplying binomials, but when students are faced with (x + 2)(x + y + 3), they will frequently just ignore the y-term.

The same can be true for factoring. It is best to avoid spending too much time with quadratics that do not have an a-value in the standard form of the quadratic ax2 + bx + c. Without a coefficient to the x^2 term, students develop the "rule" for factoring that they need to find factors that add to equal the b-term and multiply to equal the c-term. Providing students with quadratics that have a coefficient to the x^2 term early on will help them see this is a rule that is only true when a = 1. Also, providing students with opportunities to multiply binomials that have coefficients of x other than 1 is also a good practice i.e., (2x – 5)(3x + 1).

STANDARD A-SSE.B.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A. Factor a quadratic expression to reveal the zeros of the function it defines.

B. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

C. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Big Idea

Using properties of expressions, students are able to rewrite expressions in order to purposefully reveal information about the context or to generalize situations.

Cluster

Write expressions in equivalent forms to solve problems.

GRADE-LEVEL STANDARD UNPACKED

Academic Vocabulary

- **Approximate** — A value that is very close but not exactly the same as another number
- **Complete the square** — The process of converting a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides
- **Equivalent expressions** — Expressions that have the same value but are presented in different formats using the properties of numbers
- **Exponential function** — A function of the form $y = a \cdot bx$ where $a > 0$ and either $0 < b < 1$ or $b > 1$
- **Expression** — Numbers, symbols grouped together to show the value of something

Standard Cognitive Complexity

Content Complexity Rating: Level 2

Standard Progression

High School Coherence: A-SSE.2, A-SSE.4, N-RN.2

Mathematical Practices and Suggested Questions to Develop Mathematical Thinking

MP2

Reason abstractly and quantitatively

- How can we visualize a representation of completing the square?

MP4

Model with mathematics

- Given the formula for interest compounded monthly, how can I determine the APR?
- How can I convert the equation of half to determine how much of the substance will be remaining after 2 years?

A.SSE
A.SSE.A.1
A.SSE.A.2
A.SSE.B.3
A.SSE.B.4
A.ARP
A.APR.A.1
A.APR.B.2
A.APR.B.3
A.APR.C.4
A.APR.C.5
A.APR.D.6
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A.CED
A.CED.A.1
A.CED.A.2
A.CED.A.3
A.CED.A.4
A.REI
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A.REI.C.7
A.REI.C.8
A.REI.C.9
A.REI.D.10
A.REI.D.11
A.REI.D.12
TRADITIONAL APPENDIX A
INTEGRATED APPENDIX A

MP7

Look for and make use of structure

- Students must be able to take the information from a context and then decontextualize it in order to manipulate it algebraically to a different form. Once it is in a different form, students can again tie this back to the context to reveal new information.

Essential Question Stems

- In a falling object model, what form of an equation will determine when the object hits the ground?

Aspects of Rigor	Instructional Targets
Conceptual Understanding	• Given a quadratic function, explain the meaning of the zeros of the function. That is if f(x) = (x – c) (x – a), then f(a) = 0 and f(c) = 0. (DOK 2) • Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression (x –a) (x – c), a and c correspond to the x-intercepts (if a and c are real). (DOK 2) • Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay. (DOK 2)
Procedural Skills and Fluency	• Write expressions in equivalent forms by factoring to find the zeros of a quadratic function. (DOK 2) • Write expressions in equivalent forms by completing the square to convey the vertex form to find the maximum or minimum value of a quadratic function. (DOK 2)
Application	• Explain the meaning of the zeros in an expression. (DOK 2) • Explain the meaning of the vertex in a quadratic function. (DOK 2)

Suggested Instructional Strategies

On level

In Algebra 1, students should be fully engaged in the quadratic component of this standard. In connection with A.SSE.2, students should be completing the square to find a vertex, factoring to find a zero, and writing in standard form and revealing the y-intercept. Students should be looking at a lot of contextual examples, including falling object, maximizing profit, and real-life scenarios of rectangular area. Algebra 1 students may also look at exponential expressions that have integer exponents.

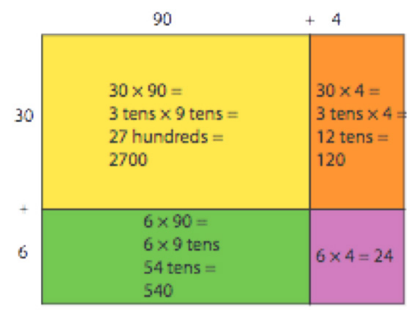
Algebra 2 students should be able to use rational exponents to recognize equivalence.

A common application of this is when students have to write equivalent expressions for compounded interest to reveal the monthly, yearly, or quarterly interest rates.

Intervention

Students may benefit from using algebra tiles to see how expressions are equivalent. Handheld manipulatives or online versions (https://illuminations.nctm.org/activity.aspx?id=3482) of algebra tiles can be useful for students to see how the factored form is equivalent to both the standard form and the vertex form of a quadratic. Algebra tiles can be related back to the area model that students use in elementary grades:

Illustrating partial products with an area model



Source: http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf

Enrichment

N/A

Ideas to Support the Standard

The standards avoid mentioning simplification, because different forms of an expression might be more appropriate and therefore more "simple" depending on the work that the student is doing with it. As this standard references, the important mathematics is about finding an equivalent expression that reveals appropriate information.

EXPLANATIONS AND EXAMPLES

The profit that a company makes selling an item (in thousands of dollars) depends on the price of the item (in dollars). If p is the price of the item, then three equivalent forms for the profit are:

Standard form: — 2p2 + 24p - 54
Factored form: — 2(p-3)(p-9)
Vertex form: — 2(p-6)2+18

Which form is most useful for finding?

a. The prices that give a profit of zero dollars?
b. The profit when the price is zero?
c. The price that gives the maximum profit?

Solution: a. Factored form, b. Standard form, c. Vertex form

Source: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434

Algebra 2

Four physicists describe the amount of a radioactive substance, Q in grams, left after t years:

a. $Q = 300e^{-0.0577t}$

b. $Q = 300(1/2)^{t/12}$

c. $Q = 300 \cdot 0.9439^{t}$

d. $Q = 252.290 \cdot 0.9439^{t-3}$

(i) Show that the expressions describing the radioactive substance are all equivalent (using appropriate rounding).

(ii) What aspect of the decay of the substance does each of the formulas highlight?

Solution:

(i) Using properties of exponents we can transform the expressions that describe the amount of the radioactive substance into each other. We have:

$$300e^{-0.0577t} = 300(e^{-0.0577})^{t} = 300 \cdot 0.9439^{t}$$

Similarly:

$$300 \cdot (1/2)^{t/12} = 300((1/2)^{1/12})^{t} = 300 \cdot 0.9439^{t}$$

Finally:

$$252.290 \cdot 0.9439^{t-3} = 252.290 \cdot 0.9439^{-3} \cdot 0.9439^{t} = 300 \cdot 0.9439^{t}$$

(ii) The first three formulas show that the initial amount of the substance is 300 grams.

a. This formula lets us read off the fact that the continuous decay rate is 5.77 %. (Note: The substance decays at a rate that is proportional to the amount present at any time and the constant of proportionality is 0.0577.)

b. If we substitute $t = 12$ we get $Q = 300 \cdot (1/2)$. Therefore, this formula shows that the half-life of the substance is 12 years.

c. Since $1 - 0.9439 = 0.0561$ we see from this formula that the annual decay rate is 5.61 %.

d. In addition to the annual decay rate, this formula also shows that when $t = 3$ we have $Q = 252.290$. This means that after 3 year there are 252.290 grams of the substance left.

Source: https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/1305

Common Misconception

Students will frequently think that the last number in an expression is the y-intercept, even if the equation is not in standard form. As students are developing understanding, continue to have them find the y-intercept by substituting in x = 0. This will help students realize that in exponential equations and in non-standard form polynomials, the last number is not necessarily the y-intercept.